

Exam. Code : 211004

Subject Code : 4644

M.Sc. (Mathematics) 4th Semester

OPERATIONS RESEARCH—II

Paper—MATH-588

Time Allowed—Two Hours] [Maximum Marks—100

Note :—Attempt any **FOUR** questions. All questions carry equal marks.

1. (a) Obtain the distribution of waiting time of a customer in the system.
- (b) On an average 96 patients per 24 hours day require the service of an emergency clinic. Also on the average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes and that each minute of decrease in the average time would cost Rs. 10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $\frac{4}{3}$ patients to $\frac{1}{2}$ a patient ?

2. (a) Derive the formula for the average number of customers in the system for the model $(M/M/1) : (N/FIFO)$.
- (b) Assume that trains are coming in a yard at the rate of 30 per day and suppose that the inter-arrival times follow exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines and one is reserved for shunting purposes), then
- (i) Calculate the probability that the yard is empty.
- (ii) Find the average queue length.
3. (a) Obtain the expression for the minimum cost for the problem of EOQ with finite rate of replenishment when shortages are not allowed.
- (b) A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that when he starts a production run, he can produce 25,000 bearings per day. The cost of holding a bearing in the stock for one year is 20 paise and the set up cost of a production run is Rs. 180. How frequently should the production runs be made ?

4. (a) Describe the method of purchase inventory problem with one price break.
- (b) Find the optimum order quantity for a product for which the price breaks are :

Quantity	unit cost (Rs.)
$0 \leq Q_1 < 500$	10.00
$500 \leq Q_2$	9.25

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering is Rs. 100.

5. (a) Explain the replacement policy when the value of money changes with time.
- (b) A manufacturer is offered two machines A and B. Machine A is priced Rs. 5,000 and running costs are estimated at Rs. 800 for each of the first five years, increasing by Rs. 200 per year in the 6th and subsequent years. Machine B, which has the same capacity as A, costs Rs. 2,500 but will have running costs of Rs. 1,200 per year for the 6th year, increasing by Rs. 200 per year thereafter. If the money is worth 10% per year, which machine should be purchased ?

6. At time zero, all items in a system are new. Each time has a probability p of failing immediately before the end of the first month of life and a probability $q = 1 - p$ of failing immediately before the end of the second month that is all items fail at the end of second month. If all the items are replaced as they fail, show that the expected number of failures $f(x)$ at the end of the month x is given by

$$f(x) = \frac{N}{1+q} [1 - (-q)^{x+1}]$$

where N is the number of items in the system.

If the cost per item of individual replacement is C_1 , and the cost per item of group replacement is C_2 , find the condition under which a group replacement policy at the end of each month is the most profitable.

7. (a) Explain in detail the concept of event type simulation.
- (b) The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous, the rain distribution is :

Event	Probability
No rain	0.50
1 cm. rain	0.25
2 cm. rain	0.15
3 cm. rain	0.05
4 cm. rain	0.03
5 cm. rain	0.02

If it did not rained on the previous day, the rain distribution is :

Event	Probability
No rain	0.75
1 cm. rain	0.15
2 cm. rain	0.06
3 cm. rain	0.04

Simulate city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random number for simulation :

69 63 39 55 29 78 70 06 78 76

Assume that for the first day of simulation it had not rained the day before.

8. A company has a single service station which has the following characteristics :

The mean arrival rate of the customers and the mean service time are 6.2 minutes and 5.5 minutes

respectively. The time between an arrival and its service varies from 1 minute to 7 minutes. The arrival and service time distributions are given below :

Time (minutes)	Probability of Arrival	Probability of Service
1-2	0.05	0.10
2-3	0.20	0.20
3-4	0.35	0.40
4-5	0.25	0.20
5-6	0.10	0.10
6-7	0.05	—

The queueing process starts at 11.00 a.m. closes at 12.00. An arrival moves immediately into the service, if the facility is empty. On the other hand, if the service station is busy, the arrival will wait in the queue. Customers are served on the first come, first served basis. If the clerk's wages are Rs. 6 per day and the customer's waiting line costs Rs. 5 per hour, would it be economical for the manager to engage the second clerk ? Use Monto-Carlo simulation technique.